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Power-law Scaling and Probabilistic Forecasting of Tsunami Runup Heights

STEPHEN M. BURROUGHS[1] and SARAH F. TEBBENS[2,3]

Abstract—A power-law scaling relationship describes tsunami runup heights at ten locations in Japan. Knowledge of the scaling law for tsunamis can be the basis for probabilistic forecasting of the size and number of future events and for estimating probabilities of extremely large events. Using tsunami runup data archived by the U.S. National Geophysical Data Center, we study ten locations where the tsunami record spans at least one order of magnitude in runup height and the temporal record extends back several decades. A power law or upper-truncated power law describes the cumulative frequency-size distribution of tsunami runup heights at all ten locations. Where the record is sufficient to examine shorter time intervals within the record, the scaling relationship for the shorter time intervals is consistent with the scaling relationship for the entire record. The scaling relationship is used to determine recurrence intervals for tsunami runup heights at each location. In addition to the tsunami record used to determine the scaling relationship, at some of the locations a record of large events (> 5 m) extends back several centuries. We find that the recurrence intervals of these large events are consistent with the frequency predicted from the more recent record. For tsunami prone locations where a scaling relationship is determined, the predicted recurrence intervals may be useful for planning by coastal engineers and emergency management agencies.

Key words: Power-law scaling, Forecasting, Tsunami runup heights, Japan

Introduction

Past studies have pursued scaling relationships to describe tsunami runup heights. Such scaling relationships may provide the basis for forecasting these infrequent, although sometimes devastating, natural disasters. A review of tsunami prediction studies, with emphasis on the United States, is provided in MOFJELD et al. (1999). ABE (1995) reported a scaling relationship between the magnitude of the source earthquake and the resultant tsunami runup. CHOI et al. (2002) reported on the scaling behavior of tsunami runup for single events measured at several sites along a coast. For Kamchatka, where some sites have only one reported event, PELINOVSKY

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[1] The University of Tampa, Department of Chemistry and Physics, Tampa, FL 33606, U.S.A.
E-mail: sburroughs@ut.edu

[2] University of South Florida, College of Marine Science, St. Petersburg, FL 33701, U.S.A.
E-mail: tebbens@marine.usf.edu

[3] Now at: Wright State University, Department of Physics, Dayton, OH 45435 U.S.A.
E-mail: sarah.tebbens@wright.edu

(1989) used an exponential scaling relationship to determine the probability of future tsunami events. Many tsunami risk assessment studies focus on hydrodynamic numerical models to simulate wave runups caused by seismic events and compare the simulations to observed runup heights (e.g., CHOI et al., 2003; HEBERT et al., 2001). RIKITAKE and AIDA (1988) studied tsunami hazard probability in Japan by evaluating the probability of tsunami-generating earthquakes and generating numerical models of associated tsunami wave heights along the coast.

An alternative approach is to examine the past record of tsunamis at individual locations to determine whether a scaling relationship describes the record. Analysis of the tsunami record at individual locations avoids differences in coastline shape and offshore bathymetry which may influence tsunami runup height at different locations along a shoreline. Probabilistic forecasting of the recurrence interval of events of a given size can be based on the frequency-size distribution of past events. Frequency-size distributions for tsunami runup heights follow a power law at Tofino, Canada (WIGEN, 1983), Miyako, Japan (OKADA and TADA, 1983), and Sanriku, Japan (HORIKAWA and SHUTO, 1983). We examine the cumulative frequency-size distributions and determine the scaling relationships and recurrence intervals for tsunami runup heights at ten locations in Japan.

Scaling Relationships

Power Law. Many natural systems have power-law cumulative frequency-size distributions. Examples include earthquakes (e.g., GUTENBERG and RICHTER, 1949), forest fire areas (MALAMUD et al., 1998), floods (MALAMUD et al., 1996; TURCOTTE and GREENE, 1993), landslide areas (TURCOTTE, 1999), rock fragmentation (TURCOTTE, 1997), and hotspot seamount volumes (BURROUGHS, 2001; TEBBENS et al., 2001). A power law applied to a cumulative distribution has the form

$$\dot{N}(r) = Cr^{-\alpha}, \tag{1}$$

where $\dot{N}(r)$ is the number of objects per unit time with size greater than or equal to r, α is the scaling exponent, and C is the activity level, a constant equal to the number of objects with size $r \geq 1$.

Upper-truncated Power Law. An upper-truncated power law has been found to describe cumulative distributions associated with several natural systems (BURROUGHS and TEBBENS, 2001a, b, 2002; TEBBENS et al., 2001). An upper-truncated power law, $\dot{N}_T(r)$ has the form

$$\dot{N}_T(r) = C\left(r^{-\alpha} - r_T^{-\alpha}\right) \tag{2}$$

where $\dot{N}_T(r)$ is the number of objects per unit time with size greater than or equal to r, r_T is the object size where the upper-truncated power law equals zero, and α is the scaling exponent. Since each value in a cumulative distribution includes all larger objects, upper truncation of the distribution decreases the cumulative number associated with each object size. In equation (2), the second term, $Cr_T^{-\alpha}$, represents this decrease from the power law, $Cr^{-\alpha}$.

Data

Tsunami events are rare enough that the number of tsunamis recorded at each site is limited. This scarcity of data has hindered the determination of scaling relationships and recurrence probabilities for tsunamis at individual locations. Tsunami size is measured as runup height, which may be measured either on a tide gauge or by the vertical height that the wave reaches on land. Using tsunami runup data archived by the U.S. National Geophysical Data Center (NGDC), we identify several locations in Japan where the tsunami record spans at least one order of magnitude in runup height and the temporal record extends back several decades.

In the NGDC dataset, tsunami runup heights are reported to the nearest 0.1 m and are therefore effectively binned in 0.1 m intervals. Thus, the actual runup height was within ± 0.05 m of the reported value. For cumulative distributions, the cumulative value for each bin represents all values of the size assigned to the bin and larger. Since each bin starts 0.05 m below the reported runup height, the reported value minus 0.05 m is assigned to the runup height for analysis of the cumulative frequency-size distributions. Further discussion on analysis and binning of cumulative distributions can be found in MALAMUD and TURCOTTE (1998), BONNET et al. (2001) and BURROUGHS and TEBBENS (2001a,b).

Analysis

We analyze cumulative frequency-size distributions of tsunami runup heights for ten locations in Japan (Fig. 1). Several functions were considered for each distribution including the power law, upper-truncated power law, and exponential function. We identified the function with the minimum chi-squared value and the best goodness of fit as determined by the Levenberg-Marquardt algorithm (PRESS et al., 2001). Each distribution is best described by a power law (Figs 2–6) or an upper-truncated power law (Fig. 7). For power-law distributions, we find the parameters C and α, equation (1), by using the Levenberg-Marquardt algorithm to minimize chi-squared (PRESS et al., 2001). This method assumes that the errors in fitting the function to the data points are normally distributed. For the distributions that are well-described by an upper-truncated power law, equation (2), we find the parameters C, α, and r_T using the same algorithm. For each parameter, the errors reported are one standard deviation (Figs. 2–7). Each cumulative frequency-size distribution is displayed with two vertical axes. The left axis is the cumulative number of tsunamis divided by the length of the data set in years, $\dot{N}$. Using this axis

Figure 1
Map of Japan showing study locations.

we can determine the number of tsunamis expected per year equal to or greater than a given height. The right axis is the recurrence interval in years, which is the reciprocal of $\dot{N}$. This axis represents the probable number of years between tsunamis equal to or greater than a given height.

Discussion

Power-law Scaling Relationship. The cumulative frequency-size distributions follow a power law for tsunami runups recorded at Choshi, Hachijo Island, Hiroo, and Mera, Japan (Fig. 2). For each of these sites, the distributions were better described by a power law than an upper-truncated power law or exponential function. Since a power law is scale invariant, the scaling relationship can be used for probabilistic forecasting of the recurrence interval of future events within the range of runup heights observed. If the same power law applies to larger runup heights, then extrapolation will predict the recurrence interval of larger events as demonstrated for two sites below.

Probabilistic Forecasting. A major objective of determining the scaling relationship for natural hazards is to use the results for probabilistic forecasting. We identify two criteria that can be used to determine whether the observed tsunami scaling

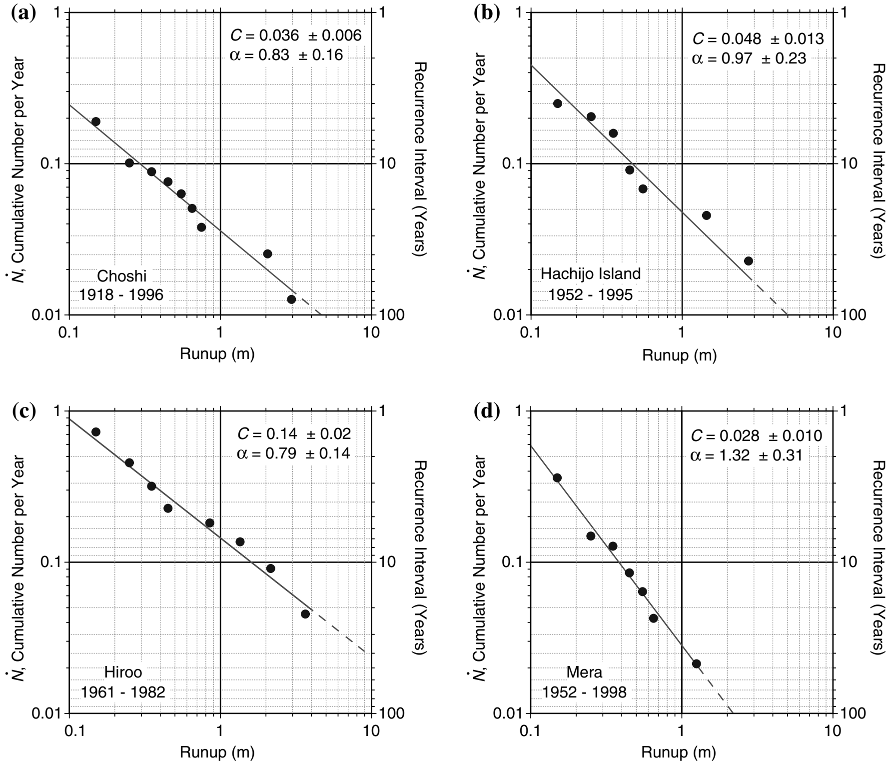

Figure 2
Cumulative frequency-size distributions for tsunami runup heights recorded in the indicated time intervals at (a) Choshi, (b) Hachijo Island, (c) Hiroo, and (d) Mera. All distributions are well-described by a power law (equation 1). The errors reported are one standard deviation.

relationship may be appropriate for probabilistic forecasting. First, the scaling exponent should be the same for both short and long time intervals. Second, the extrapolated scaling relationship yields a recurrence interval for large events that should be consistent with any large events in the record. We test these two criteria at additional locations in Japan.

Criterion 1. The record at Ayukawa is sufficiently long to examine the scaling relationship for a short time interval within the record. Small events ($<$ 0.50 m) are more common in the record after 1950, suggesting that some small events were not detected and/or recorded earlier in the historical record (Fig. 3a). To obtain the scaling relationship for the entire Ayukawa record, 1896 through 1998, we include only events with runup heights greater than 0.50 m. The cumulative frequency-size distribution for the entire record is well described by a power law (Fig. 3b). To examine the scaling relationship for a shorter time interval within the record, we examine the cumulative frequency-size distribution for events that occurred from

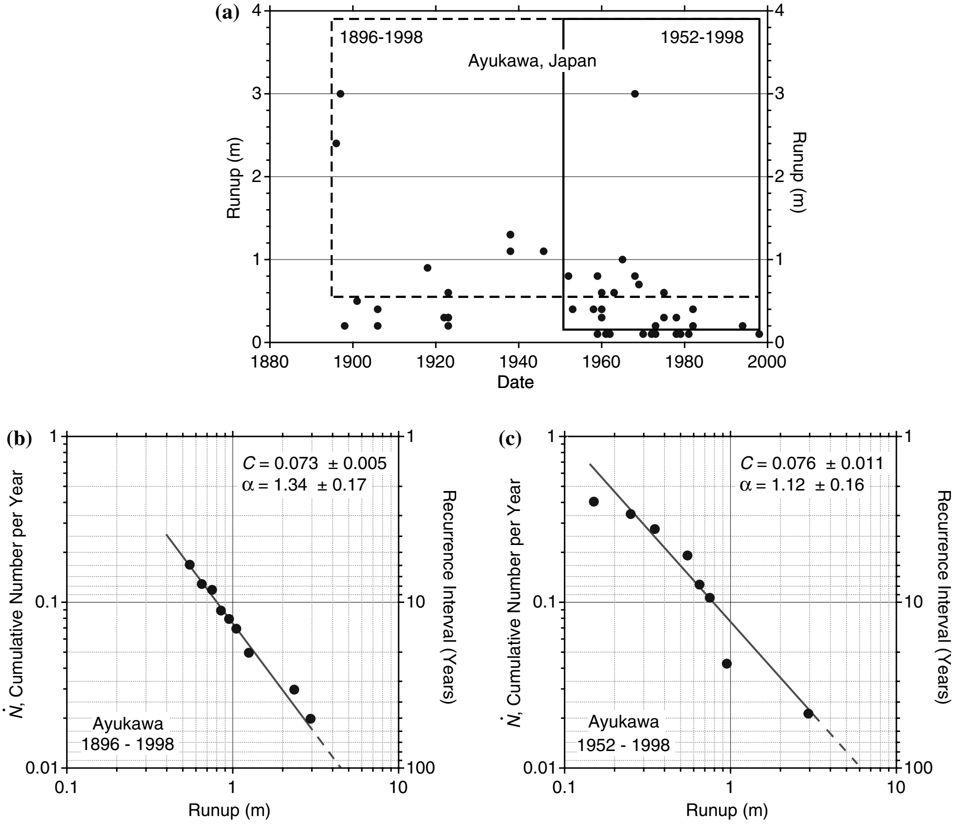

Figure 3
(a) Tsunami runup heights reported for Ayukawa, Japan. Small events (< 0.50 m) are more common in the record after 1950. Events in the dashed box (1896–1998) and in the solid box (1952–1998) are analyzed in Figures 3b and 3c, respectively. (b) Cumulative frequency-size distribution for runup heights greater than 0.50 m that occurred from 1896 through 1998. The cumulative distribution is well described by a power law (equation 1). (c) Cumulative frequency-size distribution for runup heights greater than 0.10 m that occurred from 1952 through 1998. The scaling relationship determined from the longer and shorter records are in agreement with the reported errors of one standard deviation.

1952 through 1998 (Fig. 3c). For this more recent time interval, the record of events smaller than 0.50 m appears more complete and all runup heights of 0.20 m and greater are included in the analysis. The cumulative frequency-size distribution for the shorter record is also well described by a power law (Fig. 3c). The values of α and C determined from the entire record are consistent with the values found for the shorter time interval within the reported errors of one standard deviation. Agreement of the scaling relationships for different time intervals means that the scaling relationship may be appropriate for probabilistic forecasting.

We perform a similar analysis on the Hakodate record to test criterion 1. There are few recorded events at Hakodate prior to 1950 (Fig. 4a). We examine the

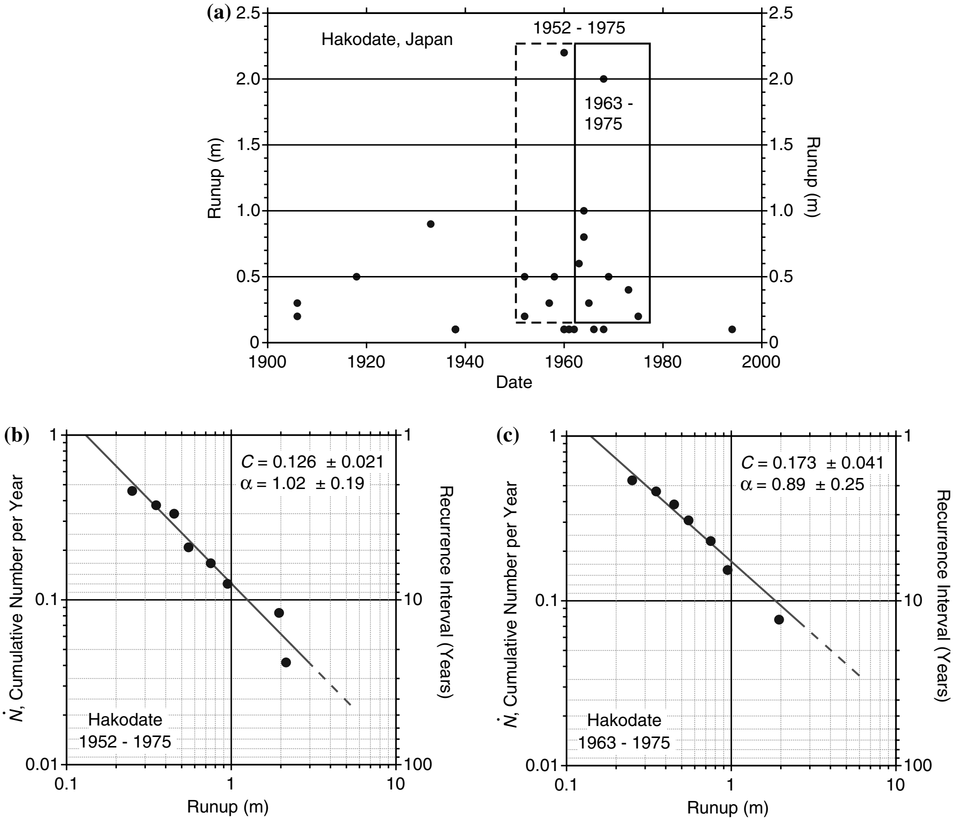

Figure 4

(a) Tsunami runup heights reported for Hakodate, Japan. Events in the dashed box (1952–1975) and in the solid box (1963–1975) are analyzed in Figures 4b and 4c respectively. (b) Cumulative frequency-size distribution for runup heights greater than 0.20 m that occurred from 1952 through 1975. The cumulative distribution is well described by a power law (equation 1). (c) Cumulative frequency-size distribution for runup heights greater than 0.20 m that occurred from 1963 through 1975. The scaling relationship determined from the longer and shorter records are in agreement with the reported errors of one standard deviation.

cumulative frequency-size distribution for the time interval from 1952 though 1975 (Fig. 4b) and for a subset of this interval from 1963 through 1975 (Fig. 4c). Events smaller than 0.3 m are not included in either analysis. The scaling relationships determined for Hakodate are consistent, within one standard deviation, for the longer and shorter time intervals.

The results at Ayukawa and Hakodate meet the first criterion that the scaling relationship should be the same for both long and short time intervals. Thus, the observed power-law scaling relationship may be appropriate for making probabilistic forecasts at each analyzed location.

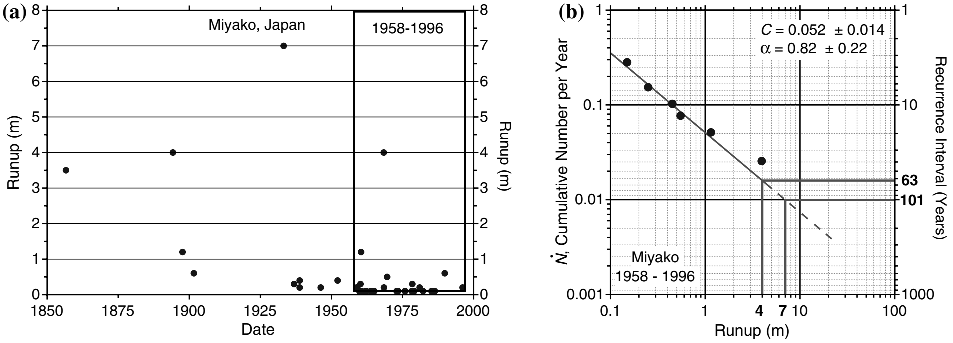

Figure 5

(a) Tsunami runup heights reported for Miyako, Japan. Events in the solid box (1958–1996) are analyzed in Figure 5b. (b) Cumulative frequency-size distribution for runup heights greater than 0.1 m that occurred from 1958 through 1996. The cumulative distribution is well described by a power law (equation 1). Based on an extrapolation of the observed scaling relationship, an event greater than or equal to 4 m is expected every 63 years and an event greater than or equal to 7 m is expected every 101 years (heavy solid lines). The historical record of 141 years contains three events greater than or equal to 4 m and one event of 7 m, consistent with these results.

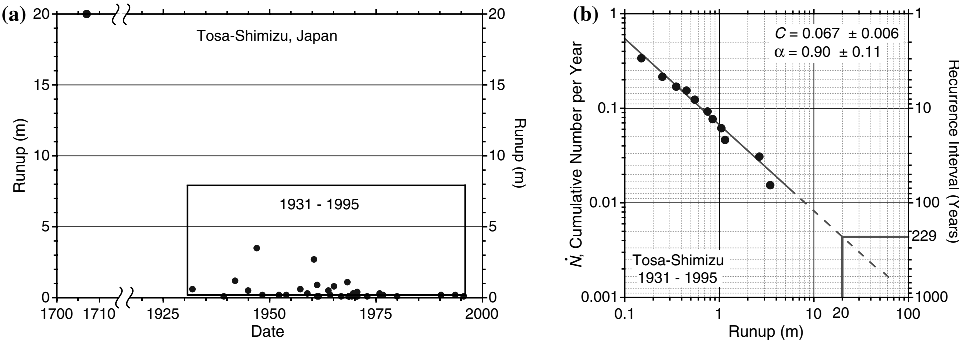

Figure 6

(a) Tsunami runup heights reported for Tosa-Shimizu, Japan. With one exception in 1707, all reported events occurred after 1930. Events in the solid box (1931–1995) are analyzed in Figure 6b. (b) The cumulative frequency-size distribution for runup heights greater than 0.1 m from 1931 through 1995. The cumulative distribution is well described by a power law (equation 1). If this power law is extrapolated to larger events, an event greater than or equal to 20 m is expected every 229 years (heavy solid lines). One 20 m event is observed in the nearly 300 year record, consistent with the extrapolated power law.

Criterion 2. At Miyako and Tosa-Shimizu there are large infrequent events in the historical record that can be compared to the scaling relationship extrapolated from the more complete recent record. For Miyako, all recorded tsunami runup heights are shown in Figure 5a. The record for events smaller than one meter is more complete after 1957. We therefore examine the scaling relationship for the interval from 1958 through 1996. We find the cumulative frequency-size distribution for

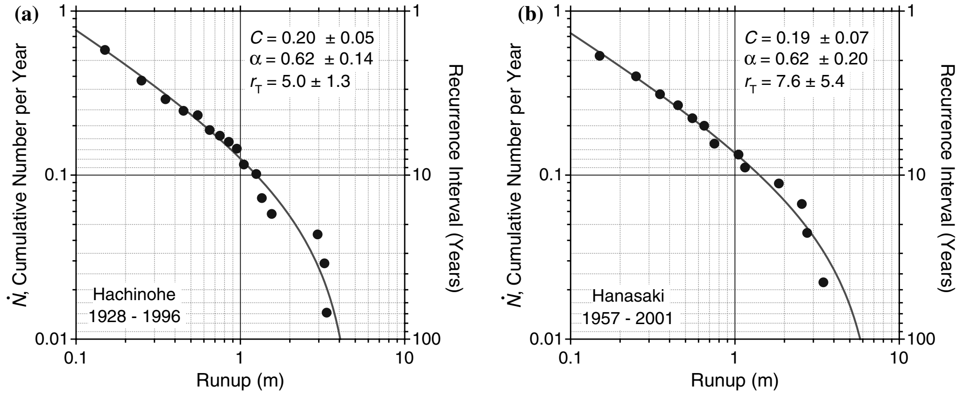

Figure 7
Cumulative frequency-size distributions for events 0.2 m and larger at Hachinohe and Hanasaki. The errors reported are one standard deviation. Both distributions are well-described by an upper-truncated power law (equation 2).

runup heights of 0.2 m and greater is well described by a power law (Fig. 5b). Prior to 1958, the record includes two relatively large events of 4 m and 7 m runup height. Based on extrapolation of the determined scaling relationship, an event greater than or equal to 4 m is expected every 63 years and an event greater than or equal to 7 m is expected every 101 years. The observed record of 141 years contains three events greater than or equal to 4 m and one event of 7 m, consistent with the expected frequency. Okada and Tada (1983) reported tsunamis larger than 5 m and 10 m at Miyako are likely once per 110 years and 250 years, respectively. Our analysis predicts slightly shorter recurrence intervals for these runup heights (76 and 136 years, respectively).

At Tosa-Shimizu a large event of 20 m runup was recorded in 1707 and all other recorded events occurred after 1930 (Fig. 6a). The cumulative frequency-size distribution for the events from 1931 through 1995 with runup heights 0.2 m and larger is well described by a power law (Fig. 6b). If this power law is extrapolated to larger runup heights, a runup greater than or equal to 20 m is expected every 229 years (Fig. 6b). One 20 m event is observed in the nearly 300 year record, consistent with this result. For both Miyako and Tosa-Shimizu, the scaling relationship extrapolated from small events predicts a recurrence interval for large events consistent with the frequency of large events in the historical record.

Upper-truncated Power Law. At Hachinohe and Hanasaki the tsunami cumulative frequency-size distributions are better described by an upper-truncated power law than by a power law or exponential function (Fig. 7). The upper-truncated power law had the minimum chi-squared value and the best goodness of fit. There are several ways to interpret the upper-truncated power law (Burroughs and Tebbens, 2001b).

The upper truncation may be a temporal effect. If this is correct, then the largest events that can occur in the region have not occurred in the historical record. This temporal truncation effect is seen for earthquake frequency-magnitude distributions when the historical record is sampled between large events (BURROUGHS and TEBBENS, 2002). Alternatively, there may be physical characteristics of a given location that influence tsunami size. Possibilities include coastline shape, seawall configuration, or offshore bathymetry. At present, we cannot discern whether the upper-truncation observed at some locations is a temporal effect, is due to local physical conditions, or is a combination of these and/or other factors.

Recurrence Intervals. The scaling relationship determined for each location provides probable recurrence intervals for tsunami runup heights (Figs. 2–7). Runup heights of one meter or greater have recurrence intervals from 5.0 to 36 years for the ten sites analyzed (Table 1). For the two sites where an upper-truncated power law describes the distribution, two recurrence intervals are given for each site (Table 1). If upper-truncation is a temporal effect, then the appropriate function for probabilistic forecasting is a power law with C and α values determined from the upper-truncated power law. If upper-truncation is caused by a physical characteristic of the site, then the upper-truncated power law is appropriate for probabilistic forecasting. The power law yields a shorter recurrence interval than the upper-truncated power law. Both recurrence intervals are given for Hachinohe and Hanasaki (Table 1).

Table 1

Scaling parameters and recurrence intervals

Location	Dates	Number of years	Number of events analyzed	C	α	Recurrence intervals for 1 m runup (years)	r_T (m)
Choshi	1918–1996	79	15	0.036	0.83	28	
Hachijo Island	1952–1995	44	11	0.048	0.97	21	
Hiroo	1961–1982	22	16	0.14	0.79	7.1	
Mera	1952–1998	47	17	0.028	1.32	36	
Ayukawa	1896–1998	103	17	0.073	1.34	14	
Ayukawa subset	1952–1998	47	19	0.076	1.12	13	
Hakodate	1952–1975	24	11	0.126	1.02	7.9	
Hakodate subset	1963–1975	13	7	0.173	0.89	5.8	
Miyako	1958–1996	39	11	0.052	0.82	19	
Tosa-Shimizu	1931–1995	65	22	0.067	0.90	15	
Hachinohe	1928–1996	69	40	0.20	0.62	5.0, 7.9	5.0
Hanasaki	1957–2001	45	24	0.19	0.62	5.3, 7.4	7.6

Conclusions

A power law or upper-truncated power law scaling relationship is observed for the cumulative frequency-size distribution of tsunami runup heights for all ten locations (Figs. 2–7). Tsunami frequency-size distributions for short time intervals are consistent with the scaling relationship found for longer intervals at the same site (Figs. 3 and 4). For sites with an historical record of large events, the frequency of these events is consistent with the recurrence interval predicted by extrapolating the scaling relationship determined from smaller events (Figs. 5 and 6). For the ten sites analyzed, the recurrence interval for a one meter event ranges from just over seven years for Hiroo to twenty-eight years for Choshi (Table 1). Knowledge of the scaling law for tsunamis can be the basis for probabilistic forecasting of the size and number of future events and for estimating probabilities of extremely large events.

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